Lumped Lossy Circuit Synthesis and Its Application in Broad-Band FET Amplifier Design in MMIC's

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Abstract —**In this paper, a lumped lossy circuit is synthesized by means of the transformation introduced in [1]. The circuit contains two different kinds of lossy branches, in which arbitrary nonuniform reactive resistors, as well as lossy inductors and capacitors, are included. This new approach can synthesize a lumped lossy matching network more flexibly than ever before. An example is presented to show the application of the synthesis of the lumped lossy matching networks in the design of a broad-band microwave integrated FET amplifier, and the advantages of the new technique can be clearly seen by comparison with the amplifier designed by the method in [1].**

I. Introduction

In the design of broad-band microwave amplifiers, lossy circuits may be better than lossless circuits as matching networks for at least four reasons. First, lossy matching networks may be able to provide a smaller *VSWR* over the required frequency range than lossless matching networks. Second, the transducer power gain (*TPG*) and the *VSWR* of a lossless matching system such as a broad-band amplifier are not independently controlled by means of lossless matching networks. That is to say that if the *TPG* is to be decreased in a certain frequency range for compensating for device gain roll-off with frequency, the *VSWR* will automatically increase in accordance with the characteristic of the unitary matrix of the lossless networks. Third, if the same quality of match is needed, the lossy matching networks might have simpler topologies than the lossless matching networks. Fourth, lossy match amplifiers exhibit stability factors superior to those of lossless match amplifiers.

Therefore, designers have never abandoned the attempt to find approaches to synthesizing and/or designing lossy matching networks. In 1953, LaRosa [2] introduced a general theory of wide-band matching with dissipative 4-poles. From the restrictions on the scattering parameters of a matching network, he cleverly found the limitation on the performance of any matching network in a simple form. On the other hand, Riddle [3] and certain others paid more attention to analytic design methods for lossy gain-compensating networks and their practical uses in the design of broad-band amplifiers, even though their methods are not generally applicable to all configurations. In 1985, a graphical design method for lossy and lossless gain-compensating networks was introduced by Villar [4]. He made use of a set of constant $|S|_{ii}$ circles plotted on a Smith chart to design broad-band lossy match amplifiers.

It is easy to see that the authors mentioned above all avoided deriving expressions for the *TPG*, the *VSWR*, and certain other characteristics of the circuits from the scattering parameters of the lossy networks directly! The reason is that, in contrast to the lossless scattering parameters, which have the characteristic of a unitary matrix, designers are not able to find limitations on the lossy scattering parameters. Thus, a general method for synthesizing and/or designing all possible lossy networks has not yet been found.

In the application of the transformation between lossy and lossless matching networks presented in [1], the present authors introduce a new technique which can easily synthesize a lumped lossy matching network more flexibly than ever before. The lumped lossy matching network synthesized here contains two different kinds of lossy branches, which comprise arbitrary nonuniform reactive resistors, and lossy inductors and capacitors. An example is given to show the design of a broad-band monolithic microwave integrated FET amplifier with lumped lossy circuits synthesized as matching networks for compensating for device gain roll-off with frequency. It can be predicted that the method described here will gain even wider applications in network design and synthesis, as well as in certain related fields.

II. Synthesis of Lumped Lossy Broad-Band Matching Networks

Lumped networks may be divided into the following four types. The first well-known lossless network, LN1 (i.e., *LCT* network), comprises inductors, capacitors, and ideal transformers. The second lumped network, LN2, consists of inductors, capacitors, resistors, and ideal transformers. It is generally called an *RLCT* network. The third type, LN3, is composed of ideal transformers and of arbitrary nonuniform lossy inductors and capacitors. It has been discussed in [1]. The last type, LN4, includes not only ideal transformers and arbitrary nonuniform lossy inductors and capacitors, but also arbitrary nonuniform reactive resistors!

The most important problem in synthesizing a lossy matching network is finding general restrictions on the parameters of a lossy matching network! This, however, is too difficult. If N is lossless, the well-known Belevitch representation [5] may be employed to obtain the unit normalized scattering parameters of the network. If N is dissipative, only the inequality derived from the concept of average power absorption remains, and no simple representation for scattering parameters can be utilized to realize the lossy network. However, this does not mean that no lossy matching networks can be synthesized. By careful consideration, we found that the second and fourth types of lumped lossy matching networks, LN2 and LN4, with no more than two kinds of lossy branches may also be synthesized by the transformation given in [1]. This is because the condition in the theorem in [1] is that "Each element of a lossy or lossless network N produces an individual impedance proportional to Z_1 or Z_2, which may be an input impedance of a lossy network."

If the above condition is satisfied by some lossy network N, it can be transformed to a lossless network M with Z_1 and Z_2 corresponding to the lossless inductor and capacitor respectively. The following algorithm will give a clear description of the synthesis of the lumped lossy matching networks.

A. Branch Impedances

According to Darlington's theory, an impedance $\tilde{Z}(s)$, if it is physically realizable, may be synthesized as a lossless network terminated with a load impedance $Z_L = 1\ \Omega$. This lossless network M, if it has transmission zeros only at zero or points at infinity, may be completely specified by the following unit normalized

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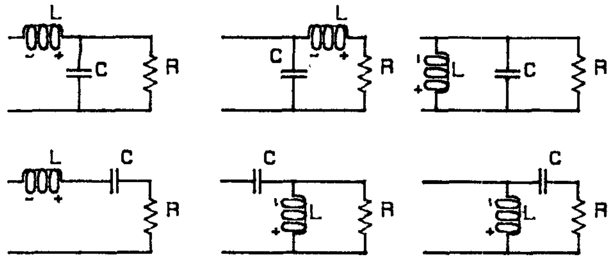

Fig. 1 The possible topologies synthesized from $\tilde{Z}_{b1}(s)$ or $\tilde{Z}_{b2}(s)$.

TABLE I
MEASURED SCATTERING PARAMETERS OF UNPACKAGED GaAs FET

FREQ. GHz	S_{11} MAG	S_{11} DEG	S_{12} MAG	S_{12} DEG	S_{21} MAG	S_{21} DEG	S_{22} MAG	S_{22} DEG
2.00	.950	-18.	.022	80.	1.91	162.	.610	-7.
2.50	.940	-24.	.025	74.	1.89	155.	.620	-12.
3.00	.930	-30.	.027	68.	1.87	149.	.630	-17.
3.50	.920	-36.	.029	63.	1.85	143.	.630	-21.
4.00	.910	-41.	.032	59.	1.82	137.	.630	-25.

input reflection factor, $\tilde{e}_{11}(s)$, and by the assumptions of Yarman and Carlin [6]:

$$\tilde{e}_{11}(s) = \frac{h(s)}{g(s)} = \frac{h_1 + h_2 s + \cdots + h_{n+1}s^n}{g_1 + g_2 s + \cdots + g_{n+1}s^n} \tag{1}$$

where $h(s)$ and $g(s)$ are the numerator and denominator polynomials of $\tilde{e}_{11}(s)$ with the complex angular frequency s as the variable. Since $\tilde{Z}(s)$ contains a resistor, it may be considered an input impedance of LN2.

However, the lossless network M can be transformed to a lossy network N (LN3) by

$$\tilde{E} = \{(I+E) - \sqrt{Z_1 Z_2}(I-E)\}\{(I+E) + \sqrt{Z_1 Z_2}(I-E)\}^{-1} \tag{2a}$$

$$E = \{\sqrt{Z_1 Z_2}(I+\tilde{E}) - (I-\tilde{E})\}\{\sqrt{Z_1 Z_2}(I+\tilde{E}) + (I-\tilde{E})\}^{-1} \tag{2b}$$

where Z_1 and Z_2 denote the arbitrary nonuniform lossy inductor and capacitor respectively [1], and $\tilde{E}$ and E are the scattering matrices corresponding to the lossless and lossy networks. With this transformation the input impedance $Z(s)$ of the lossy network N will become an input impedance of LN4 when the lossy network is terminated with a reactive resistor Z_L, which is given as [7]

$$Z_L(j\omega) = R_L Z_r(j\omega) = R_L \omega_n^{1/2} / \{1 + [jQ_r \omega_n^{3/2}]/3\}. \tag{3}$$

Here, $\omega_n = \omega/\omega_m$ is the normalized angular frequency. Q_r, the so-called quality factor of the reactive resistor, is defined as

$$Q_r = \omega_m C_T R_{ac} \tag{4}$$

where ω_m is the angular frequency at which Q_r and the ac resistor R_{ac} are measured; C_T is the parasitic shunt capacitance.

B. Possible Topologies of the Branch Impedances

If $\tilde{Z}(s)$ is used as the branch impedance Z_1 or Z_2 described in theorem in [1] and denoted $\tilde{Z}_{b1}$ or $\tilde{Z}_{b2}$, then the corresponding topology will not be too complex for practical applications. Thus, the maximum degree of the numerator or denominator n_{b1} or n_{b2} will be specified as not greater than 2. The possible topologies synthesized from $\tilde{Z}_{b1}$ or $\tilde{Z}_{b2}$ are shown in Fig. 1.

If the inductor and the capacitor are of arbitrary nonuniform lossy type [1] and the resistor is of arbitrary nonuniform reactive type, the corresponding branch impedances Z_{b1} and Z_{b2} of LN4 may be obtained by substituting $Z(s)$ for Z_1 and Z_2 in terms of the theorem in [1].

C. Supposed Lossless Matching Network

A lossless matching network M is assumed and its unit normalized reflection factor has the same expression as (1). If the same topologies of the branch impedances are not needed, n, which specifies the maximum degree of the scattering parameters of the supposed lossless matching network, will not exceed 2.

D. Transformation from the Supposed Lossless Matching Network to a Lossy One

The transformation is performed again with (2) in order to obtain the scattering parameters $e_{i,j}(s)$ $(i, j = 1,2)$ of the lumped lossy matching network N from the supposed lumped lossless matching network M.

It should be emphasized in the above transformation that the type of branch impedances, Z_{b1} and Z_{b2} or $\tilde{Z}_{b1}$ and $\tilde{Z}_{b2}$, selected to substitute for Z_1 and Z_2 in (2) will depend on the type of branch circuits, LN2 or LN4, needed.

E. Optimization

The expression for the *TPG* of a lossy match amplifier [1] is employed. Input and output *VSWR*'s are derived from the scattering parameters of the cascaded lossy matching networks and FET's. After optimization, better scattering parameters $\tilde{e}_{11,i}(s)$ $(i = 1, NM)$ of the supposed lossless matching networks can be obtained, where *NM* is the number of matching networks.

F. Synthesis of Lumped Lossy Matching Networks

i) The branch impedances $\tilde{Z}_{b1}$ and $\tilde{Z}_{b2}$ are synthesized first. If the branch circuits are of the LN4 type, the inductors, capacitors, and 1 Ω resistors obtained by continuous fraction expansions of $\tilde{Z}_{b1}$ and $\tilde{Z}_{b2}$ will be replaced by arbitrary nonuniform lossy inductors, capacitors, and reactive resistors respectively. Thus, Z_{b1} and Z_{b2} are achieved indirectly. ii) In synthesis of the input impedance $\tilde{Z}(s)$, which corresponds to certain $\tilde{e}_{11,i}(s)$ $(i = 1,2,\cdots, NM)$, the "inductances" and "capacitances" obtained should be considered real positive multiplicative constants of $\tilde{Z}_{b1}$ and $\tilde{Z}_{b2}$ or Z_{b1} and Z_{b2}. Hence, they need not be antinormalized with respect to the frequency. iii) Finally, elements in the branch circuits are weighted by the real positive multiplicative constants individually.

III. COMPUTER-AIDED SYNTHESIS

As an application of the above approach, the following example is presented to illustrate the synthesis of the lumped lossy matching networks in the design of a one-stage broad-band monolithic microwave integrated FET amplifier. The measured FET scattering parameters (Table I) are used over the band 2–4 GHz. It is found that in this specified passband, the magnitude of S_{11} is larger and the device is unstable with unilateral conjugate gain from 17.75 dB to 15.04 dB. The input matching network is constructed by the cascaded matching networks LN3 and LN4. The lossy network LN4 is mainly applied for compensating for the device gain roll-off with frequency and stabilizing the amplifier. At the output terminal of the FET, only the matching network LN3 is used. The element models in (2a) and

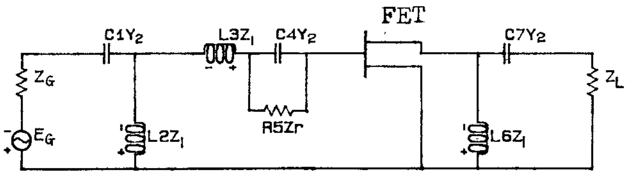

Fig. 2 Design of one-stage amplifier: $C1 = 0.9473$ pF, $L2 = 5.512$ nH, $L3 = 7.192$ nH, $C4 = 0.8511$ pF, $R5 = 61.23$ Ω, $L6 = 3.994$ nH, $C7 = 1.159$ pF

(2b) [1] and in (3) are employed and the inputs are summarized as follows.

- Generator: $Z_G = 50$ Ω. Load: $Z_L = 50$ Ω.
- Skin loss of the inductors: $Q_L = 25$.
- Skin loss of the capacitors: $Q_c = 2000$.
- Dielectric loss of the capacitors: $Q_d = 30$.
- Quality factor of the reactive resistor: $Q_r = 0.01$.
- Frequency at which the above losses are measured: $f_m = 4.2$ GHz.
- Lossy element models:

$$Z_1 = \left(j\omega + 6497.92\omega^{1/2} \right)$$

$$Y_2 = 1/Z_2 = 1/\left(1/j\omega + \omega^{1/2}/8.5738\times10^{18} + 1/30\omega\right)$$

$$Z_r = \omega_n^{1/2}/\left\{1 + \left[j0.01\omega_n^{3/2} \right]/3\right\}.$$

- Passband: 2 GHz $\leqslant f \leqslant$ 4 GHz.
- Maximum complexity of the matching networks:
  - Input matching network I (LN3): $n = 2$, $k = 2$.
  - Input matching network II (LN4): $n = 1$, $k = 0$.
    - Maximum complexity of the network (LN4) corresponding to the branch impedance Z_{b1}: $n_{b1} = 2$, $k_{b1} = 0$; No branch impedance Z_{b2}, for n is equal to 1.
  - Output matching network (LN3): $n = 2$, $k = 2$.

The performance of the amplifier to be optimized is given below. Note that the device can only provide a gain of about 9 dB at 4 GHz with stability.

- Flat gain level: $T_0(\omega) = 9.0$ dB.
- Input and output $VSWR$'s: $VSWR_{in} = 1.5$; $VSWR_{out} = 1.5$.

A. Results of Optimization

The supposed lossless input reflection factors corresponding to input matching networks I (LN3) and II (LN4) are

$$\tilde{e}_{11,1}(s) = \frac{0.1375 + 0.2282s}{0.1375 + 0.5719s + s^2}$$

$$\tilde{e}_{11,2}(s) = \frac{0.6123s}{1 + 0.6123s}.$$

The supposed lossless input reflection factor corresponding to branch I (LN4) is

$$\tilde{e}_{11,b1}(s) = \frac{0.8622s + 2.1315s^2}{1 + 2.2375s + 2.1315s^2}.$$

The supposed lossless input reflection factor corresponding to the output matching network (LN3) is

$$\tilde{e}_{11,3}(s) = \frac{-0.1551 + 0.0899s}{0.1551 + 0.5642s + s^2}.$$

The configuration and the performance of the better amplifier are shown in Fig. 2 and Fig. 3.

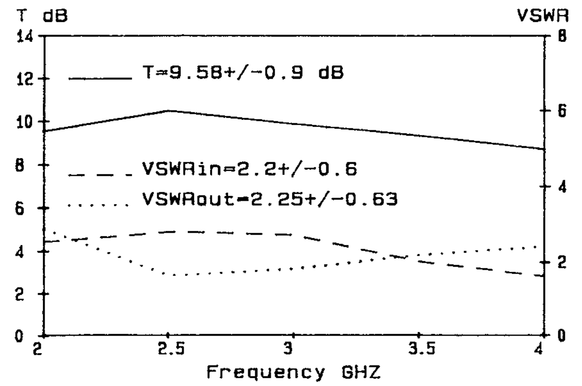

Fig 3. Performance of the optimized amplifier.

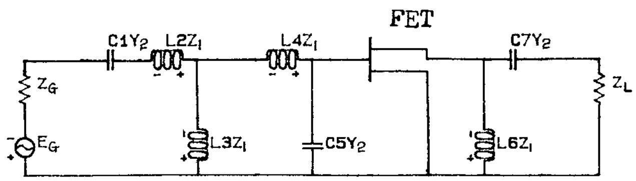

Fig 4 One-stage amplifier designed by SLMNA: $C1 = 0.6342$ pF, $L2 = 2.671$ nH, $L3 = 4.398$ nH, $L4 = 5.578$ nH, $C5 = 0.033$ pF, $L6 = 5.544$ nH, $C7 = 1.101$ pF

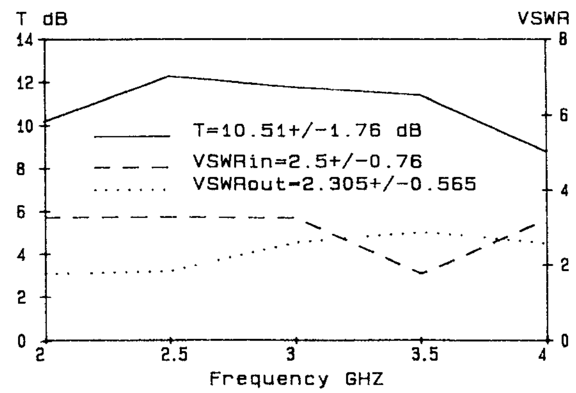

Fig 5 Performance of the amplifier designed by SLMNA

The advantages of the technique mentioned above can be illustrated through a comparison with the following one-stage amplifier designed by the method described in [1]. The amplifier has the same load and generator conditions, element models, passband, and goal performances as the above example. The other inputs to the program SLMNA are given below.

The maximum complexity of the matching networks is as follows.

Input matching network (LN3): $n = 4$, $k = 2$.

Output matching network (LN3): $n = 2$, $k = 2$.

The supposed lossless input reflection factors corresponding to the input and output matching networks (LN3) are

$$\tilde{e}_{11,1}(s) = \frac{0.2574 + 1.1400s - 0.2715s^2 + 3.0210s^3 + 0.1337s^4}{0.2574 + 1.5710s + 1.9970s^2 + 3.1200s^3 + 0.1337s^4}$$

$$\tilde{e}_{11,2}(s) = \frac{-0.1177 + 0.1734s}{0.1177 + 0.5152s + s^2}.$$

The configuration and the performance of the better amplifier designed by the program SLMNA are illustrated by Fig. 4 and Fig. 5.

B. Discussion

On condition that the above two examples can be easily and practically manufactured, their performances are compared with each other. It is clear that both have the same number of matching elements. However, the former has a greater stability factor, $K_{min} = 2.61$, than the latter, $K_{min} = 1.36$ (K_{min} being the minimum stability factor over the passband), and has smaller input and output *VSWR*'s and a smoother *TPG*, even though its *TPG* is 0.93 dB less than that of the latter on the average. Thus it can be seen that it is worth choosing lumped lossy circuits as matching networks.

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